

chth; 0; at d

vè; k; 12

12.1 Hkkfedk

ge $x + 3$, $y \in 5, 4x + 5, 10y \in 5]$ bR; kfn t\$ s l jy chth; 0; at d kls i ffpfr gks pøø gSA
d{k k vi e} geus n{kk Fkk fd ;s 0; at d fd l izdkj igfy; k v k leL; kvka dks , d
l q; ofLFkr izdkj ls itr djuseal gk; d gksrgSA ge l jy lehdj. kka okys vè; k; eHkh
0; at d klo vud mnkgj. kka dks n{kk pøø gSA

chtxf.kr e@0; at d k (expressions) dks, d oñh; vo/kj.kk ekuk tkrk gSA ;g vè; k;
chth; 0; at d kls l c½ gkk A tc vki bl vè; k; dks i <+y> rks vki dks Kkr gks tk, xk
fd chth; 0; at d fd l izdkj curs g\$ blg fd l izdkj l a kstr fd; k (feyk; k) tkrk g\$
buoø eku ge oñl s Kkr dj l drs g\$ rFkk budk fd l izdkj mi ;kx fd; k tk l drk gSA

12.2 0; at d fd l izdkj curs g\$

vc ge Hkyh Hkkfr tkurs g\$ fd , d pj (variable) D; k gksrk gSA ge pjka dks 0; Dr djus
oñ fy,] v{kjk x, y, l, m, ... bR; kfn dk i z kx djrs gSA , d pj oñ fofHkklu eku gks l drs
gSA bl dk eku fuf' pr ughagksrk gSA bl oñ nñjh v k v pj (constant) dk , d fuf' pr eku
gksrk gSA vpjka oñ mnkgj. k 4]100]&17] bR; kfn gSA

ge pjka v k v pjka dks l a kstr djoø chth; 0; at d k dks cukrs gSA bl oñ fy,
ge ; kx] 0; odyu] xqku v k foHkk tu dh l fØ; kvka dk i z kx djrs gSA ge]
4x + 5, 10y \in 20 t\$ s 0; at d k dks i gys gh n{kk pøø gSA 0; at d 4x + 5, -x Ø pj oñ i z kx
l s cuk g\$ ft l e s i gys pj x dks v pj 4 l s xqkk djoø v k fi Ø bl xqkui Ø eavpj 5
t kM+dj i klr fd; k tk rk gSA bl h izdkj] 10y \in 20 i gys pj y dks v pj 10 l s xqkk djoø
v k fi Ø bl xqkui Ø e a l s 20 ?V k dj i klr fd; k tk rk gSA

mi jkDr 0; at d pjka v k v pjka dks l a kstr djoø i klr fd, x, FksA ge 0; at d k dks
pjka dks lo; amu pjka l s v Fkok v U; pjka l s l a kstr djoø Hkh i klr dj l drs gSA

- nf[k, fd fuEufyf[kr 0; $\frac{1}{2}d$ fdl idkj ikr fd, tkrs g\\$\\
 $x^2, 2y^2, 3x^2 \text{ ó } 5, xy, 4xy + 7$
- (i) 0; $\frac{1}{2}d$ x^2 pj x dks Lo; ax Is xqkk djo& ikr fd; k tkrk g\\$A
 $\sqrt{Fkz} - x \times x = x^2$
 $ftl idkj 4 \times 4 = 4^2$ fy[kk tkrk g\\$ ml h idkj ge $x \times x = x^2$. fy[krs g\\$A bl s
I kekl; r% x dk oxl (x squared) i< k tkrk g\\$A
1ckn e] tc vki ?kkrkd vks?kkr* okysvè; k; dk vè; ; u djks rc vki vukko
djs fd x^2 dks x o& mlij ?kkr 2 Hkh i< tk l drk g\\$A
bl h idkj] ge fy[k l drs g\\$ % $x \times x \times x = x^3$
I kekl; r% x^3 dks x dk ?ku (x cubed) i< tk rk g\\$A ckn e] vki ; g vukko djs
fd x^3 dks x o& mlij ?kkr 3 Hkh i< tk l drk g\\$A
 x, x^2, x^3, \dots e] Is i& d x Is ikr, d cth; 0; $\frac{1}{2}d$ g\\$A
- (ii) 0; $\frac{1}{2}d$ $2y^2$ dks y Is bl idkj ikr fd; k tkrk g\\$ $2y^2 = 2 \times y \times y$
; g\\$ ge y dks y Is xqkk djo& y^2 ikr djs g\\$ vks fi0j bl xqkuiy y^2 dks 2 Is
xqkk djs g\\$A
- (iii) $(3x^2 \text{ ó } 5)$ e] ge igys x^2 ikr djs g\\$ vks fi0j ml s 3 Is xqkk djo& $3x^2$ ikr djs
g\\$A vr e] $3x^2 \text{ ó } 5$ ij igpus o& fy,] ge $3x^2$ e] Is 5 dks ?kVkr g\\$A

i& kl dhft,



crkb, fd fuEufyf[kr
0; $\frac{1}{2}d$ fdl idkj ikr
fd, tkrs g\\$A

$7xy + 5, x^2y, 4x^2 \text{ ó } 5x$

- (iv) xy e] ge pj x dks, d vll; pj y Is xqkk djs g\\$A bl idkj]
 $x \times y = xy$
- (v) $4xy + 7$ e] ge igys xy ikr djs g\\$ ml s 4 Is xqkk djo& $4xy$
ikr djs g\\$ vks fi0j fn; k gvk 0; $\frac{1}{2}d$ ikr djs o& fy,]
 $4xy + 7$ tkMfs g\\$A

12.3 , d 0; $\frac{1}{2}d$ o& in

vHkh rd mlij geus i< g\\$ fd 0; $\frac{1}{2}d$ fdl idkj cuk, tkrs g\\$ vc ge ml s, d
l q; ofLFkr : i eaj[kksA bl dk; Zo& fy,] ge; g tkuus dh vko'; drk g\\$fd, d 0; $\frac{1}{2}d$
o& in (terms) vks muo& xqku[KM (factors) D; k gks g\\$ vFkz~muo& vFkz D; k g\\$A
0; $\frac{1}{2}d$ ($4x + 5$) ij fopkj dhft, A bl 0; $\frac{1}{2}d$ dks cukus o& fy,] igys geus vyx Is
4 vks x dk xqkk djo& $4x$ cuk; k Fkk vks fi0j bl e] 5 tkM+fn; k Fkk A bl h idkj] 0; $\frac{1}{2}d$
($3x^2 + 7y$) ij fopkj dhft, A ; g\\$ geus igys vyx Is 3, x vks x dk xqkk djo& $3x^2$ cuk; k
FkkA fi0j geus vyx Is 7 vks y dk xqkk djo& $7y$ cuk; k Fkk A $3x^2$ vks $7y$ cukus o& ckn]
geus fn; k gvk 0; $\frac{1}{2}d$ ikr djs o& fy,] budks tkM+fn; k Fkk A
vki ik, xsfd ge ftrus Hkh 0; $\frac{1}{2}d$ i j dk; Zdjs g\\$os I Hkh bl h : i eans{ks tk l drs
g\\$A buo& Hkkx gks g\\$ tks vyx Is cuk, tkrs g\\$ vks fi0j tkM+fn, tkrs g\\$ 0; $\frac{1}{2}d$ o& bl
idkj o& Hkkx] tks igys vyx Is cuk, tkrs g\\$ vks fi0j tkM+fn, tkrs g\\$ bl 0; $\frac{1}{2}d$ o&
in dgys g\\$A 0; $\frac{1}{2}d$ $4x^2 \text{ ó } 3xy$ dks nf[k, A ge dgrs g\\$fd bl o& nks in $4x^2$ vks $63xy$
g\\$A in $4x^2$; 4, x vks x dk xqkuiy g\\$ rFkk in $63xy$; $63, x$ vks y dk xqkuiy g\\$A

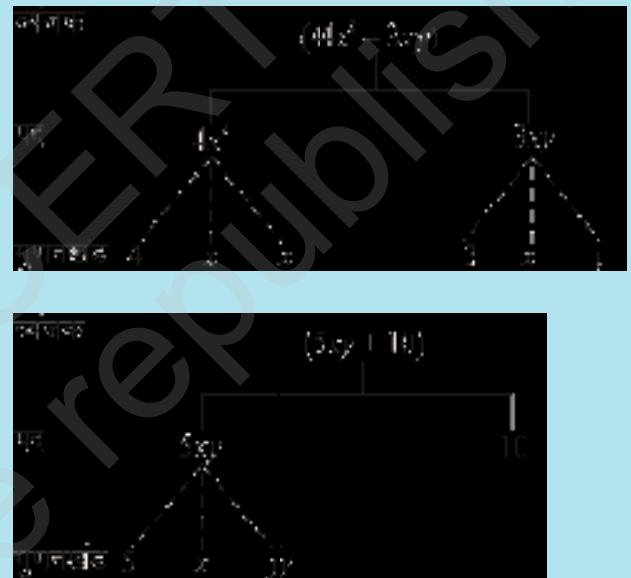
0; atd dks cukus o \varnothing fy, i nka dks tkM \varnothing tkrk gSA ftl i zdkj 0; atd (4x + 5) dks cukus o \varnothing fy, 4x v \varnothing 5 dks tkM \varnothing tkrk g \varnothing ml h i zdkj 0; atd (4x 2 ó 3xy) dks cukus o \varnothing fy, 4x 2 v \varnothing (ó 3xy) dks tkM \varnothing tkrk gSA bl dk dkj.k 4x 2 + (ó 3xy) = 4x 2 ó 3xy gksk gSA

é;ku nhft, fd in eaÍ.k (minus) fpÉ I fefyf gksk gSA 0; atd 4x 2 ó 3xy
el geus in dks 3xy u y \varnothing (ó 3xy) fy; k Fkk A bl fy,] gea ;g dgus fd
vko'; drk ughagSfd ,d 0; atd dks cukus o \varnothing fy,] i nka dks tkM \varnothing ; k ?kvk; k tkrk
gSA bl o \varnothing fy, o \varnothing oy ;g dguk gh i ;k \varnothing r g \varnothing fd i nka dks tkM \varnothing tkrk gSA

, d in o \varnothing xqku[kM

geus mQij n \varnothing kk Fkk fd 0; atd (4x 2 ó 3xy) o \varnothing nks
in 4x 2 v \varnothing ó 3xy gSA in 4x 2 ; 4, x v \varnothing x dk xqkui ly
g \varnothing ge dgrs g \varnothing fd 4, x v \varnothing x in 4x 2 o \varnothing xqku[kM
(factors) gSA ,d in vius xqku[kMka dk ,d
xqkui ly gksk gSA in ó 3xy, xqku[kMka63, x v \varnothing y
dk ,d xqkui ly gSA

ge ,d 0; atd o \varnothing in ka rFkk i nka o \varnothing xqku[kMka
dks ,d lfo/ktud v \varnothing vkd"kd i zdkj ls ,d
0; atd i M+v \varnothing j \varnothing k (tree diagram) }jkj fu: fir dj
I drs gSA 0; atd (4x 2 ó 3xy) dk i M+I \varnothing yXu v \varnothing Nfr
ean'kkz k x; k gSA



é;ku nhft, fd i M+v \varnothing j \varnothing k el geus xqku[kM o \varnothing
fy, cmfdr j \varnothing kkvka dk i z kx fd; k rFkk i nka o \varnothing
fy, Irr j \varnothing kkvka dk i z kx fd; k gSA ;g buo \varnothing
fefJr u gksus o \varnothing fy, fd; k x; k gSA

vkb, 0; atd 5xy + 10 dk i M+v \varnothing j \varnothing k [khpaA xqku[kM ,d s fy[ks tk,] fd ftuo \varnothing vks
xqku[kM u gks l o \varnothing A bl i zdkj] ge 5xy dks 5 × xy o \varnothing : i eaughafy[krs g \varnothing D; kfd xy o \varnothing
vks v \varnothing Hkh xqku[kM gks l drs g \varnothing A bl h i zdkj] ;fn x 3 , d in gksk rks bl s x × x 2 u fy[k
dj x × x × x fy[kk tk, A l kfk gh] ;kn jf[k, 1 dks vyx ls xqku[kM ughafy; k tkrk gSA

i z kl dhft,

1. fuEufyf[kr 0; atd kaeadk&dk] ls in g \varnothing \ n'kkb, fd ;s 0; atd o \varnothing scuk, tkrs gSA
i R; d 0; atd o \varnothing fy, ,d i M+v \varnothing j \varnothing k Hkh [khfp, A
8y + 3x 2 , 7mn ó 4, 2x 2 y
2. ,d s rhu 0; atd fyf[k,] ftuea ls i R; d e spkj in g \varnothing



xqkkd

ge , d in dksml o \varnothing xqku[kmka o \varnothing , d xqkui \varnothing y o \varnothing : i e \varnothing fy[kuk 1 h[k p \varnothing gSA buea ls , d xqku[km 1 \varnothing ; kRed (numerical) gks l drk gSrFkk vU; chth; (algebraic) gks l drs g \varnothing (\sqrt{Fkk} ~buea pj gks g \varnothing A bl 1 \varnothing ; kRed xqku[km dks in dk 1 \varnothing ; kRed xqkkd (numerical coefficient); k o \varnothing oy xqkkd dgrs gSA bls 'k \varnothing k in (tks Li "Vr% chth; xqku[kmka dk xqkui \varnothing y g \varnothing dk xqkkd Hkh dgrsgA bl i \varnothing dkj] in $5xy + xy$ dk xqkkd 5 gSA bl h i \varnothing dkj] in $10xyz$, e \varnothing xyz dk xqkkd 10 gSrFkk in $67x^2y^2 + x^2y^2$ dk xqkkd 67 gSA

tc fd l h in dk xqkkd +1 gksk g \varnothing ik; % mls fy[krs l e; Nkm+fn; k tkrk gSA mnkgj. kfk 1x dks x fy[kk tkrk g \varnothing 1 x^2y^2 dks x^2y^2 fy[kk tkrk g \varnothing br; kfn A l kfk gh] xqkkd (61) dks o \varnothing oy 1 .k fpE (&) l s n'kkz k tkrk gSA bl i \varnothing dkj] (&1)x dks -x fy[kk tkrk g \varnothing (61) x^2y^2 dks - x^2y^2 fy[kk tkrk g \varnothing br; kfn A

dHkh&dHkh 'kCn xqkkd dk i \varnothing ks , d vf/d 0; k i d : i e \varnothing izks fd; k tkrk gSA bl : i e \varnothing ge dgrs g \varnothing fd in $5xy + xy$ dk xqkkd 5 g \varnothing 5y dk xqkkd x gSrFkk 5x dk xqkkd y gSA $10xy^2 + xy^2$ dk xqkkd 10 g \varnothing 10y 2 dk xqkkd x gSrFkk 10x dk xqkkd y 2 gSA bl i \varnothing dkj] bls vf/d 0; k i d : i e \varnothing xqkkd , d 1 \varnothing ; kRed xqku[km gks l drk gS; k , d chth; xqku[km gks l drk gS; k nks ; k vf/d xqku[kmka dk xqkui \varnothing y Hkh gks l drk gSA bls 'k \varnothing k xqku[kmka o \varnothing xqkui \varnothing y dk xqkkd dgk tkrk gSA

i \varnothing kl dhft,

fuEufyf[kr 0; tdk e \varnothing os in NfV, tks vpj ugha gSA muo \varnothing 1 \varnothing ; kRed xqkkd Hkh fyf[k, %
4x + 3y, a + b + 5,
2y + 5, 2xy

mnkgj. k 1 fuEufyf[kr 0; tdk e \varnothing os in NfV, tks vpj ugha gSA muo \varnothing 1 \varnothing ; kRed xqkkd Hkh fyf[k, %

$$xy + 4, 13 + y^2, 13 + y + 5y^2, 4p^2q + 3pq^2 + 5$$

gy

Øe 1 \varnothing ; k	0; atd	in (tksvpj ugha g \varnothing)	1 \varnothing ; kRed xqkkd
(i)	$xy + 4$	xy	1
(ii)	$13 + y^2$	$+ y^2$	61
(iii)	$13 + y + 5y^2$	$+ y$ $+ 5y^2$	61 5
(iv)	$4p^2q + 3pq^2 + 5$	$4p^2q$ $+ 3pq^2$	4 63

mnkj. k 2

- (a) fuEufyf[kr 0; atd ea x oř D; k xqkkd gš\
 $4x \circ 3y, 8 \circ x + y, y^2x \circ y, 2z \circ 5xz$
- (b) fuEufyf[kr 0; atd ea y oř D; k xqkkd gš\
 $4x \circ 3y, 8 + yz, yz^2 + 5, my + m$

gy

- (a) iř; d 0; atd eř ge xqku[kM x okys in dks nškrs gšA ml in dk 'ksk Hkkx xdk
okřNr xqkkd gšsk A

Øe l ř; k	0; atd	xqku[kM x okyk in	x dk xqkkd
(i)	$4x \circ 3y$	$4x$	4
(ii)	$8 \circ x + y$	$\circ x$	61
(iii)	$y^2x \circ y$	y^2x	y^2
(iv)	$2z \circ 5xz$	$\circ 5xz$	$\circ 5z$

- (b) bl dh fof/ mijkDr (a) dh fof/ tř h gh gšA

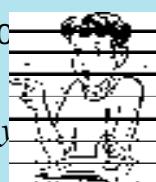
Øe l ř; k	0; atd	xqku[kM y okyk in	y dk xqkkd
(i)	$4x \circ 3y$	$\circ 3y$	63
(ii)	$8 + yz$	yz	z
(iii)	$yz^2 + 5$	yz^2	z^2
(iv)	$my + m$	my	m

12.4 Ieku vřs vleku in

tc inka oř chth; xqku[kM , d tř s gh gš rks os in Ieku in (like terms) dgykrs gš tc inka oř chth; xqku[kM fHklu&fHklu gš rks os vřl eku in (unlike terms) dgykrs gšA mnkj. kFk 0; atd $2xy \circ 3x + 5xy \circ 4$, ea inka $2xy \circ 5xy$ dks nš[k, A $2xy \circ xqku[kM 2, x \circ 5xy \circ y gšA 5xy \circ xqku[kM 5, x \circ y gšA$ bl iřkij] buoř chth; (vFkř~osftuea pj gš xqku[kM , d gh gš vřs bl hfy, ;s Ieku in gš A bl oř foijhr] inka $2xy \circ -3x$ ea fHklu&fHklu chth; xqku[kM gšA ;s vřl eku in gšA bl h iřkij] in $2xy \circ 4$ vřl eku in gšA I kFk gh] $\circ 3x \circ 4$ Hkk vřl eku in gšA

iř kl dhft,

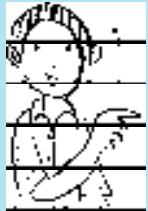
fuEufyf[kr eř Ieku inka c
I eř cukb, %
 $12x, 12, \circ 25x, \circ 25, \circ 25j$
 $1, x, 12y, y$



12.5 , dinh] f}in] f}in vřs cgj in

og chth; 0; atd ft leaooy , d in gš , dinh (monomial) dgykrk gš tř s7xy, $\circ 5m, 3z^2, 4$ bR; kfn A

iż kl dħift,



fuEufyf[kr 0; at dks dks , dñh] f} in vks f-kin oħ : i eħoxħNr dħift, %a, a + b, ab + a + b, ab + a + b ó 5, xy, xy + 5, 5x² - x + 2, 4pq ó 3q + 5p, 7, 4m ó 7n + 10, 4mn + 7.

, d 0; at d ftleħo y nks in għażiex os vleku in għaqgħ f} in (binomial) dgykrk għ mnkgj .kkFkx + y, m ó 5, mn + 4m, a² ó b² f} in għA 0; at d 10pq , d f} in uħagħġi ; g , d , dinh għA 0; at d (a + b + 5) , d f} in uħagħi għSA bl ċarhu in għA , d 0; at d ftleħarhu in għi , d f-ki n (trinomial) dgykrk għ mnkgj .kkFkx + y + 7, ab + a + b, 3x² ó 5x + 2, m + n + 10 f-ki n għA i jrrq 0; at d ab + a + b + 5, d f-ki n uħagħġi bl ċarhu in u għad-pkji in għA 0; at d x + y + 5x , d f-ki n uħagħġi SD; kif id in x vksx 5x l-eku in għA

0; ki d : i eħi , d ; k] vf/d inka okyk 0; at d , d cagħi n (Polynomial) dgykrk għA bl iċċekj] , dñh] f} in h vksx f-ki nh. Hkk cagħi n għA

mnkgj.k dkj. k Ifgr crkb, fd inka oħ fuEufyf[kr ; Nekka ead dks & dks ls ; He ieku inka oħ għar Fkk dks & dks ls ; He vleku inka oħ għA

- | | | | |
|--|---|------------------------------|--------------|
| (i) 7x, 12y | (ii) 15x, 621x | (iii) ó 4ab, 7ba | (iv) 3xy, 3x |
| (v) 6xy ² , 9x ² y | (vi) pq ² , ó 4pq ² | (vii) mn ² , 10mn | |

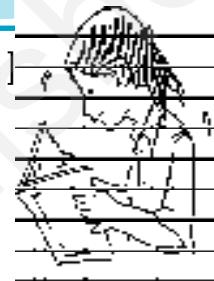
gy

Øe l- f- k	; k e	xq ku [kM	chħi; xq ku [kM , d gh għ ; k fħkluu & fħkluu għ	I eku@ vleku in	fVli . kħ
(i)	7x 12y	7, x 12, y }	fħkluu & fħkluu	vleku	inika eapj fħkluu & fħkluu għ
(ii)	15x 621x	15, x 621, x }	, d gh għ	I eku	
(iii)	ó 4ab 7 ba	ó 4, a, b 7, b, a }	, d gh għ	I eku	; kn jf[k, ab = ba
(iv)	3xy 3x	3, x, y 3, x }	fħkluu & fħkluu	vleku	pj yħoloy iġys in eaqgħi
(v)	6xy ² 9x ² y	6, x, y, y 9, x, x, y }	fħkluu & fħkluu	vleku	nksuka inika eapj rks , d tħiġi għi i jrru bux ?kkra vyx vyx vyx għ
(vi)	pq ² ó 4pq ²	1, p, q, q ó 4, p, q, q }	, d gh għ	I eku	è; ku nhift, l- xqkkad 1 fn [kk; k uħażżeek għ

fuEufyf[kr l jy pj.k vki dks ; g fu.kz yseal gk; d gksfd fn, gq in leku in g; k vleku in g%

- I ; kRed xqkkdij e;ku u nhft, A inkaol chth; Hkkx ij viuk e;ku ofnr dhft, A
- inkaespjkd dh tkp dhft, A ; s , d gh gkspkfg, A
- vcj inkaea iR; d pj dh ?krkadh tkp dhft, A ; s , d gh gksuh pkfg, A
e;ku nhft, fd leku inkaol ckjs esfu.kz yssle] bu nks crkla sdkbz iHko ugha iMfk gS% (1) inkaol I ; kRed xqkkd rFkk (2) inkaespjkaol xqkk djusdk Øe A

izukoyh 12.1



- fuEufyf[kr fLFkfr; kae pjk vpjkavkj vd xf.krh; l fØ;kvadk izkx djrs gq]
chth; 0; at d iklr dhft, %
 - I ; kyeal s z dks ?kvukA
 - I ; kvka x vkj y o; kx dk vkl/k A
 - I ; k z dksLo; amI ls xqkk fd; k tkrk gSA
 - I ; kvka p vkj q o; xqkui ly dk , d&pkfkkbZA
 - nksuka l ; kvka x vkj y o; oxk dks tkMuk tkrk gSA
 - I ; kvkam vkj n o; xqkui ly o; rhu xqseal ; k 5 tkMuk A
 - 10 eals I ; kvka y vkj z xqkui ly dks ?kvuk A
 - I ; kvka a vkj b o; xqkui ly eal s muo; kx dks ?kvuk A
- (i) fuEufyf[kr 0; at dka ea inka vkj muo; xqku [kMka dks NkfV, A inka vkj muo; xqku [kMka dks i M+vkj{kka }jkj Hkh n'kk, A
 - $x \approx 3$
 - $1 + x + x^2$
 - $y \approx y^3$
 - $5xy^2 + 7x^2y$
 - $\approx ab + 2b^2 \approx 3a^2$
(ii) uhps fn, 0; at dka ea inka vkj muo; xqku [kMka dks NkfV, A
 - $\approx 4x + 5$
 - $\approx 4x + 5y$
 - $5y + 3y^2$
 - $xy + 2x^2y^2$
 - $pq + q$
 - $1.2 ab \approx 2.4 b + 3.6 a$
(g) $\frac{3}{4}x + \frac{1}{4}$
(h) $0.1 p^2 + 0.2 q^2$
- fuEufyf[kr 0; at dka ea inkaol I ; kRed xqkkdij tksvpju gk dh igpku dhft, A
 - $5 \approx 3t^2$
 - $1 + t + t^2 + t^3$
 - $x + 2xy + 3y$
 - $100m + 1000n$
 - $\approx p^2q^2 + 7pq$
 - $1.2 a + 0.8 b$
 - $3.14 r^2$
 - $2(l+b)$
 - $0.1 y + 0.01 y^2$
- (a) os in igpku, ftuesx gS vkj fi0j bueax dk xqkkd fyf[k, A
 - $y^2x + y$
 - $13y^2 \approx 8yx$
 - $x + y + 2$
 - $5 + z + zx$
 - $1 + x + xy$
 - $12xy^2 + 25$
 - $7 + xy^2$

- (b) os in igpkfu, ftuesy² gsvkj fi0j bueay² dk xqkkd fyf[k, A
 (i) 8 ó xy² (ii) 5y² + 7x (iii) 2x²y ó 15xy² + 7y²
5. fuEufyf[kr 0; atdkas, dnt] f}in vkj fkin oI : i eaoxb0r dhft, %
 (i) 4y ó 7z (ii) y² (iii) x + y ó xy (iv) 100
 (v) ab ó a ó b (vi) 5 ó 3t (vii) 4p²q ó 4pq² (viii) 7mn
 (ix) z² ó 3z + 8 (x) a² + b² (xi) z² + z (xii) 1 + x + x²
6. crkb, fd fn, gq inka oI ;je leku inka oI gs ;k vleku inka oI gs %
 (i) 1, 100 (ii) ó7x, $\frac{5}{2}$ x (iii) ó 29x, ó 29y
 (iv) 14xy, 42yx (v) 4m²p, 4mp² (vi) 12xz, 12x²z²
7. fuEufyf[kr ea l eku inka dks NfV, %
 (a) ó xy², ó 4yx², 8x², 2xy², 7y, ó 11x², ó 100x, ó 11yx, 20x²y,
 ó 6x², y, 2xy, 3x
 (b) 10pq, 7p, 8q, ó p²q², ó 7qp, ó 100q, ó 23, 12q²p², ó 5p², 41, 2405p, 78qp,
 13p²q, qp², 701p²

12.6 chth; 0; atdkas oI ;kx vkj 0; odyu

fuEufyf[kr 1eL; kvka ij fopkj dhft, %

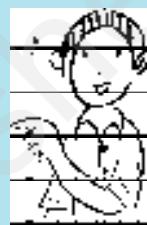
1. Ifjrk oI ikl oI N oIps gSA vehuk oI ikl ml Is 10 oIps vf/d gSA vliwdgrk gSfd ml oI ikl Ifjrk vkj vehuk oI ikl oIy ftrus oIps gSm I s 3 vf/d oIps gSA vki vliwoI oIpkadk I ;k oI s Kkr djxs\
- pfid ;g ughafn; k x; k gSfd Ifjrk oI ikl fdrus oIps gS bl fy, ge blgaxeku yss gSA vehuk oI ikl bul s 10 vf/d] vFkkz-x + 10 oIps gSA vliwdgrk gSfd ml oI ikl Ifjrk vkj vehuk oI oIy oIpkadk I s 3 vf/d oIps gSA vr% ge Ifjrk vkj vehuk oI oIpkadk ;kx Kkr djrs gS vkj ml ;kx ea 3 tkMfs gS vFkkz-x, x + 10 vkj 3 dks tkMfs gS
2. jkewoI fir k dh orZku vkJ; qjkewdh vkJ; q dh rhu xph gSA jkewoI nknkth dh vkJ; q jkewvkJ jkewoI fir k dh vkJ; qoI ;kx I s 13 o"kl vf/d gSA vki jkewoI nknkth dh vkJ; qfdI idk j Kkr djxs\
- pfid jkewdh vkJ; qnh gbjZ ugha gS bl fy, vkb, bl s y o"klZeku ya rc] ml oI fir k dh vkJ; q3y o"kl gSA jkewoI nknkth dh vkJ; qKkr djus oI fy,] geajkewdh vkJ; q(y) vkJ ml oI fir k dh vkJ; q(3y) dk ;kx Kkr djoI bl ;kx ea 13 tkMuk gksk] vFkkz-ge y, 3y vkJ 13 dk ;kx Kkr djuk i Msk A
3. , d ckx eI xykc vkJ xns oI iks' soxkdkj D; kfj; ka ea yxk, tkrs gSA ftI oxkdkj D; kjh ea xns oI iky yxk, tkrs gSm dh Hkjk dh yckbz ml oxkdkj D; kjh dh Hkjk dh yckbz I s 3 ehVj vf/d gS ftI ea xykc oI iks' yxk, x, gSA xns dh D; kjh xykc dh D; kjh I s {kski ly ea fdruh cM gS\



vkb, xykc dh D; kjh dh Hkotk dks i ehVj eku yssgSA rc xns dh D; kjh dh Hkotk (l+3) ehVj gksA buoQ {kski ly (oxZehVj e) Øe'k% l^2 vks (l+3)^2 gksA bu nkska dk vrj gh ;g crk, xk fd xns oQ ikskaokyh D; kjh xykcksaokyh D; kjh l s {kski ly ea fdruh cM gSA

mi jkDr rhuk fLFkfr; ke] geachth; 0; atdkadks tkMuk ;k ?kvuk i M Fkk A nsud thou e] bl h idkj dh vusd , h fLFkfr; gekjs l Ee[k vkrh g] tgk geachth; 0; atdkadk i zks djuk i M Fkk gS rFkk mu ij vdxfr.krh; l fO; k, i djuh i M Fkk g] bl vuPNn e] ge ;g nsks fd chth; 0; atdkadks fall i idkj tkMuk vks ?kvk; k tkrk gSA

i zkl dhft,



de l sde , h nksfLFkfr; ka oQ ckjses l ksp, ftue l s i R; d eavki dks nkschth; 0; atdkadks cukus dh vko'; drk i M vks mla gksa tkMuk ;k ?kvuk i M gSA

I eku i nka dk tkMuk vks ?kvuk

I jyre 0; atd , dnh gks gSA bueaooy , d gh in gkrk gSA i kjk dju soQ fy,] ge ;g I h[ks fd I eku i nka dks fd l i idkj tkMuk ;k ?kvk; k tkrk gSA

- vkb, 3x vks 4x. dks tkMuk ge tkurs gfd x , d l q; k gS rFkk bl hfy, 3x vks 4x Hkh l q; k, i gSA.

$$\text{VC, } 3x + 4x = (3 \times x) + (4 \times x)$$

$$= (3 + 4) \times x$$

$$= 7 \times x = 7x$$

$$;k \quad 3x + 4x = 7x$$

forj.k ;k cWu xqk oQ i zks l s

pfd pj] l q; k, i gh g] bl fy, ge forj.k xqk dk i zks dj l drs g]

- vkb, vc vks 8xy, 4xy vks 2xy dks tkMuk

$$\begin{aligned} 8xy + 4xy + 2xy &= (8 \times xy) + (4 \times xy) + (2 \times xy) \\ &= (8 + 4 + 2) \times xy \\ &= 14 \times xy = 14xy \end{aligned}$$

$$;k \quad 8xy + 4xy + 2xy = 14xy$$

- vkb, 7n eal s 4n dks ?kvk, A

$$\begin{aligned} 7n \circ 4n &= (7 \times n) \circ (4 \times n) \\ &= (7 \circ 4) \times n = 3 \times n = 3n \end{aligned}$$

$$;k \quad 7n \circ 4n = 3n$$

- bl h i idkj] 11ab eal s 5ab dks ?kvk, A

$$11ab \circ 5ab = (11 \circ 5) ab = 6ab$$

bl h i idkj] nks ;k vf/d l eku i nka dk ;ks , d l eku in gkrk g] ft l dk l q; k red xqkka l Hkh l eku i nka oQ xqkka oQ ;ks oQ cjkj gkrk gSA



b1 h i₁dkj] nks l eku i₁ndk d₁ v₁r₁j , d₁ l eku i₁n g₁rk g₁ ft₁ dk₁ l₁; k₁ed x₁kk₁ nks uka l eku i₁nk₁ o₁ l₁; k₁ed x₁kk₁ o₁ v₁r₁ o₁ cjk₁ g₁rk g₁SA

è; ku nhft , fd v₁l eku i₁nk₁ dks ml₁ i₁dkj t₁km₁; k₁ ?kv₁k; k₁ ugh₁ tk₁ d₁rk₁] ft₁ i₁dkj fd₁ l₁ eku i₁nk₁ dks t₁km₁; k₁ ?kv₁k fy; k₁ t₁kr₁ g₁SA b₁ o₁ mnkg₁.k ge i₁gys gh₁ n₁sk p₁ø g₁SA tc₁ x e₁5 dks t₁km₁ t₁kr₁ g₁ ft₁ rks ge b₁ i₁fj.kke dks(x+5) fy [krsg₁SA è; ku nhft , fd₁ (x+5) e₁5 v₁l₁ x nks uka gh₁ i₁gys t₁ssgh g₁SA b₁ h i₁dkj₁] ; fn ge v₁l eku i₁nk₁ 3xy v₁l₁ 7 dks t₁km₁ rks ; k₁x 3xy + 7 g₁SA ; fn ge 3xy e₁ls 7 ?kv₁k, rks i₁fj.kke 3xy ó 7 g₁SA

0; ki d₁ chth; 0; at₁dk₁ dk₁ t₁km₁ v₁l₁ ?kv₁kuk

v₁kb , o₁N mnkg₁.k y₁ø%

- 3x + 11 v₁l₁ 7x ó 5 dks t₁km₁ A

$$\text{ok} \text{N}r ; kx = 3x + 11 + 7x \overset{\circ}{=} 5$$

v₁c] ge t₁kr₁ fd₁ i₁n 3x v₁l₁ 7x l₁ eku i₁n g₁rf₁kk 11 v₁l₁ ó 5 H₁h l₁ eku i₁n g₁SA l₁ k₁F₁ gh₁] 3x + 7x = 10x v₁l₁ 11 + (ó 5) = 6 g₁SA vr% ge mijk₁dr ; k₁x dks u₁hps fn , v₁u₁dkj l₁ jy d₁j l₁ drs g₁%

$$; kx = 3x + 11 + 7x \overset{\circ}{=} 5$$

$$= 3x + 7x + 11 \overset{\circ}{=} 5 \quad (\text{inkadks i} \text{ø} ; \text{bfLFkr djus ij})$$

$$= 10x + 6$$

$$vr\% 3x + 11 + 7x \overset{\circ}{=} 5 = 10x + 6$$

- 3x + 11 + 8z v₁l₁ 7x ó 5 dks t₁km₁ A

$$; kx = 3x + 11 + 8z + 7x \overset{\circ}{=} 5$$

$$= 3x + 7x + 11 \overset{\circ}{=} 5 + 8z \quad (\text{inkadks i} \text{ø} ; \text{bfLFkr djus ij})$$

è; ku nhft , fd₁ geus l₁ eku i₁nk₁ dks , d₁ l₁ k₁F₁ j₁[kk g₁rf₁kk v₁o₁y v₁l₁ eku i₁n 8z ml₁ h i₁dkj jgr₁ g₁SA

$$vr\% ; kx = 10x + 6 + 8z$$

- 3a ó b + 4 e₁ls a ó b dks ?kv₁kb , A

$$v₁r₁j = 3a ó b + 4 ó (a ó b)$$

$$= 3a ó b + 4 ó a + b$$

è; ku nhft , fd₁ fd₁ i₁dkj geusa ó b dks dksBdka e₁

j₁[kk a rF₁kk fd₁ i₁dkj dksBdka dks [kksyrs l₁e; fpÉka dk₁ è; ku j₁[kk g₁sl eku i₁nk₁ dks , d₁ l₁ k₁F₁ j₁[kusol fy ,]

i₁nk₁ dks i₁ø ; bfLFkr djus ij]

è; ku nhft , %

$$t\text{ss} \overset{\circ}{=} (5 \overset{\circ}{=} 3) = 65 + 3 \text{ g}\text{ss}$$

$$mlh i₁dkj ó(a ó b) = óa + b$$

g₁ chth; i₁nk₁ o₁ fpÉka ij

mlh i₁dkj dk₁ fd₁; k₁ t₁kr₁

g₁ t₁kr₁ fd₁ l₁; kv₁ka o₁ fpÉka

o₁ l₁ k₁F₁ fd₁; k₁ t₁kr₁ g₁

$$v₁r₁j = 3a ó a ó b + b + 4$$

$$= (3 \overset{\circ}{=} 1) a ó (1 \overset{\circ}{=} 1) b + 4$$

$$v₁r₁j = 2a + (0) b + 4 = 2a + 4$$

$$; k] 3a ó b + 4 ó (a ó b) = 2a + 4$$

vc] ge vH;kl oQ fy,] 0;at dks oQ ;kx vks 0;odyu ij oQN vks mnkj.k gy djksA

mnkj.k 4 Iku inks , df=kr djoQ 0;at d

$$12m^2 + 9m + 5m = 4m^2 + 7m + 10 \text{ dks l jy dhft, \%}$$

gy inks iQ0;bfLkr djs i j] ges i klr gksk g%

$$12m^2 + 4m^2 + 5m = 9m + 7m + 10$$

$$= (12 + 4)m^2 + (5 + 9 + 7)m + 10$$

$$= 8m^2 + (6 + 4 + 7)m + 10$$

$$= 8m^2 + (6 + 11)m + 10$$

$$= 8m^2 + 11m + 10$$

mnkj.k 5

$$30ab + 12b + 14a = 24ab + 10b + 18a \text{ dks ?Vkb, A}$$

gy

$$30ab + 12b + 14a = (24ab + 10b + 18a)$$

$$= 30ab + 12b + 14a = 24ab + 10b + 18a$$

$$= 30ab + 24ab + 12b + 10b + 14a + 18a$$

$$= 6ab + 22b + 32a$$

ofYid : i l} ge 0;at dks dks , d oQ uhps , d djoQ bl i dkj j [ks gfd Iku in , d gh lh] vFkk~Lrkkae jg] tsk uhps n'kkz k x; k g%

$$\begin{array}{r} 30ab + 12b + 14a \\ 24ab + 10b + 18a \\ \hline 6ab + 22b + 32a \end{array}$$

mnkj.k 6

$$2y^2 + 3yz, 0; y^2 + yz + z^2 \text{ vks } yz + 2z^2 \text{ oQ ;kx eis } 3y^2 + z^2$$

$$\text{vks } 0; y^2 + yz + z^2 \text{ oQ ;kx dks ?Vkb, A}$$

gy

$$igys ge 2y^2 + 3yz, 0; y^2 + yz + z^2 \text{ vks } yz + 2z^2 \text{ dks tkMfs gSA}$$

$$\begin{array}{r} 2y^2 + 3yz \\ 0; y^2 + yz + z^2 \\ \hline y^2 + 3yz + z^2 \end{array}$$

è;ku nlift, fd ,d in

?Vkusdk vFk gSfd ml oQ

;k; ifryke dks tkMukA

vr% 610b ?Vkusdk vFk gSfd + 10b tkMukA 618a

?Vkusdk vFk gSfd + 18a

tkMuk rFkk 24ab ?Vkusdk

vFk gS fd 6 24ab dks

tkMukA ?Vk, tkus okys

0;at d oQ uhps n'kkz x,

fpÉ?Vkusdk ifO;k dks

mfpr : i ls djs eis

I gk; d gks g

(1)

fiQj ge] 3y^2 + z^2 vks 0; y^2 + yz + z^2 dks tkMfs gSA

$$\begin{array}{r} 3y^2 + z^2 \\ 0; y^2 + yz + z^2 \\ \hline 2y^2 + yz \end{array}$$

(2)

i z kl dhft ,

tkMuk vks ?Vkb, %

(i) m o n, m + n

(ii) mn + 5 o 2, mn + 3



vc ge ;ks (1) e₁ l s ;ks (2) dks ?Vkr s g₁A

$$\begin{array}{r}
 y^2 + 3yz + z^2 \\
 2y^2 + yz \\
 \hline
 0 \quad 0 \\
 \hline
 0y^2 + 2yz + z^2
 \end{array}$$

izukoyh 12.2

1. Iku inkadks l₁kstr (feyk) djo₁ l jy dhft, %



- (i) $21b + 32 + 7b + 20b$
- (ii) $6z^2 + 13z^2 + 5z + 7z^3 + 15z$
- (iii) $p + (p + q) + q + (q + p)$
- (iv) $3a + 2b + ab + (a + b + ab) + 3ab + b + a$
- (v) $5x^2y + 5x^2 + 3yx^2 + 3y^2 + x^2 + y^2 + 8xy^2 + 3y^2$
- (vi) $(3y^2 + 5y + 4) + (8y + y^2 + 4)$

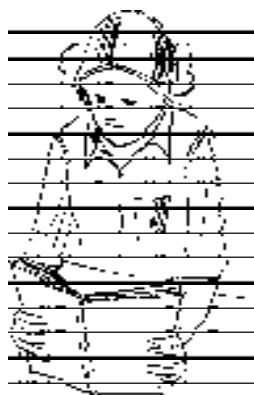
2. tkM₁ %

- (i) $3mn, + 5mn, 8mn, + 4mn$
- (ii) $t + 8tz, 3tz + z, z + t$
- (iii) $+ 7mn + 5, 12mn + 2, 9mn + 8, + 2mn + 3$
- (iv) $a + b + 3, b + a + 3, a + b + 3$
- (v) $14x + 10y + 12xy + 13, 18 + 7x + 10y + 8xy, 4xy$
- (vi) $5m + 7n, 3n + 4m + 2, 2m + 3mn + 5$
- (vii) $4x^2y, + 3xy^2, + 5xy^2, 5x^2y$
- (viii) $3p^2q^2 + 4pq + 5, + 10p^2q^2, 15 + 9pq + 7p^2q^2$
- (ix) $ab + 4a, 4b + ab, 4a + 4b$
- (x) $x^2 + y^2 + 1, y^2 + 1 + x^2, 1 + x^2 + y^2$

3. ?Vkb, %

- (i) $y^2 - e_1 l s + 5y^2$
- (ii) $612xy - e_1 l s - 6xy$
- (iii) $(a + b) - e_1 l s (a + b)$
- (iv) $b (5 + a) - e_1 l s a (b + 5)$
- (v) $4m^2 + 3mn + 8 - e_1 l s + m^2 + 5mn$
- (vi) $5x + 10 - e_1 l s + x^2 + 10x + 5$
- (vii) $3ab + 2a^2 + 2b^2 - e_1 l s + 5a^2 + 7ab + 5b^2$
- (viii) $5p^2 + 3q^2 - pq - e_1 l s + 4pq + 5q^2 + 3p^2$

4. (a) $2x^2 + 3xy + 2b^2 - e_1 l s + 5a^2 + 7ab + 5b^2$
 (b) $+ 3a + 7b + 16 - e_1 l s + 2a + 8b + 10 - e_1 l s + D; k ?Vkb plfg, \backslash$



5. ó x^2 ó y^2 + 6xy + 20 iklr djus o fy,] 3x² ó 4y² + 5xy + 20 e D; k fudky yuk pkfg, \
6. (a) 3x ó y + 11 Vó ó y ó 11 o ; kx ea l s 3x ó y ó 11 dks ?Vkb, A
 (b) 4 + 3x Vó 5 ó 4x + 2x² o ; kx ea l s 3x² ó 5x Vó óx² + 2x + 5 o ; kx dks ?Vkb, A

12.7 fdI h 0; atd dk eku Kkr djuk

ge tkurs gfd , d chth; 0; atd dk eku ml 0; atd dks cukusokys pjka o ekukaij fuHkj djrk gSA , h vuul fLFkfr; k g tgj gea 0; atdka o eku Kkr djus gks g t s fd ge ; g tkp djuk pkgrsgfd pj dk , d fo'k k eku , d fn, gq lehdj.k dks l r V djrk gS ; k ughA

tc ge T; kfefr vó ifrfnu dh xf.kr o l kdk i kx djrs g rks Hkh ge 0; atdka o eku Kkr djrs gSA mnkgj. lkFk] Hkotk zokys oxZdk {kski ly l² gsk gSA ; fn l = 5 cm g rks {kski ly 5² cm² = 25 cm² gSA ; fn Hkotk = 10 cm g rks {kski ly 10² cm² ; k 100 cm² g bR; kfn A , l s o N vó mnkgj. kka dks ge vxys vuPNn ea nksA

mnkgj.k 7 fuEufyf[kr 0; atdka o eku x = 2 o fy, Kkr dhft, %

- (i) $x + 4$ (ii) $4x \text{ ó } 3$ (iii) $19 \text{ ó } 5x^2$
 (iv) $100 \text{ ó } 10x^3$



gy

- (i) $x + 4$ e $x = 2$ j [kus ij] gea x + 4 dk fuEufyf[kr eku iklr gsk g
 $x + 4 = 2 + 4 = 6$
- (ii) $4x \text{ ó } 3$ e $x = 2$ j [kus ij] gea iklr gsk g
 $4x \text{ ó } 3 = (4 \times 2) \text{ ó } 3 = 8 \text{ ó } 3 = 5$
- (iii) $19 \text{ ó } 5x^2$ e $x = 2$ j [kus ij] gea iklr gsk g
 $19 \text{ ó } 5x^2 = 19 \text{ ó } (5 \times 2^2) = 19 \text{ ó } (5 \times 4) = 19 \text{ ó } 20 = 1$
- (v) $100 \text{ ó } 10x^3$ e $x = 2$ j [kus ij] gea iklr gsk g%
 $100 \text{ ó } 10x^3 = 100 \text{ ó } (10 \times 2^3) = 100 \text{ ó } (10 \times 8) [e; ku nhft, fd 2^3 = 8 g]$
 $= 100 \text{ ó } 80 = 20$

mnkgj.k 8 fuEufyf[kr 0; atdka o eku Kkr dhft,] tc n = ó 2

- (i) $5n \text{ ó } 2$ (ii) $5n^2 + 5n \text{ ó } 2$ (iii) $n^3 + 5n^2 + 5n \text{ ó } 2 g\%$

gy

- (i) $5n \text{ ó } 2$ e $n = ó 2$ j [kus ij] gea iklr gsk g
 $5(ó 2) \text{ ó } 2 = ó 10 \text{ ó } 2 = ó 12$
- (ii) $5n^2 + 5n \text{ ó } 2$ e $n = ó 2$ o fy,] $5n \text{ ó } 2 = ó 12 g$
 $Vó] 5n^2 = 5 \times (ó 2)^2 = 5 \times 4 = 20$ [pfd $(ó 2)^2 = 4$]

nkuska dks feykus i j] gea i klr gksk gS%

$$5n^2 + 5n \leq 2 = 20 \leq 12 = 8$$

(iii) $\sqrt{c} n = \sqrt{2} \text{ or } fy,$

$$5n^2 + 5n \leq 8 \text{ gS rFkk}$$

$$n^3 = (\leq 2)^3 = (\leq 2) \times (\leq 2) \times (\leq 2) = \leq 8 \text{ gS}$$

nkuska oF feykus i j]

$$n^3 + 5n^2 + 5n \leq 2 = \leq 8 + 8 = 0$$

vc ge nkspjka oF 0; at dk tS sx + y, xy bR; kn i j fopkj djksA nkspjka okys, d
0; at dk dk lq; kred eku Kkr djus oF fy,] gea bl ea nkuska pjka oF eku j[kus dh
vko'; drk gksrh gSA mnkgj. lkFk] x = 3 vL y = 5 oF fy, (x + y) dk eku
3 + 5 = 8 gSA

mnkgj.k 9 $a = 3$ vL $b = 2$ oF fy,] fuEufyf[kr 0; at dk oF eku Kkr dhft, %

(i) $a + b$

(ii) $7a \leq 4b$

(iii) $a^2 + 2ab + b^2$

(iv) $a^3 \leq b^3$

gy fn, gq 0; at dk eka a = 3 vL b = 2 j[kus i j] gea i klr gksk gS%

(i) $a + b = 3 + 2 = 5$

(ii) $7a \leq 4b = 7 \times 3 \leq 4 \times 2 = 21 \leq 8 = 13.$

(iii) $a^2 + 2ab + b^2 = 3^2 + 2 \times 3 \times 2 + 2^2 = 9 + 12 + 4 = 25$

(iv) $a^3 \leq b^3 = 3^3 \leq 2^3 = 3 \times 3 \times 3 \leq 2 \times 2 \times 2 = 9 \times 3 \leq 4 \times 2 = 27 \leq 8 = 19$

i zukoyh 12.3



1. ; fn m = 2 gS rks fuEufyf[kr oF eku Kkr dhft, %

(i) $m \leq 2$

(ii) $3m \leq 5$

(iii) $9 \leq 5m$

(iv) $3m^2 \leq 2m \leq 7$ (v) $\frac{5m}{2} \leq 4$

2. ; fn p = 6 2 gS rks fuEufyf[kr oF eku Kkr dhft, %

(i) $4p + 7$

(ii) $\leq 3p^2 + 4p + 7$

(iii) $\leq 2p^3 \leq 3p^2 + 4p + 7$

3. fuEufyf[kr 0; at dk oF eku Kkr dhft,] tc x = 61 gS%

(i) $2x \leq 7$

(ii) $\leq x + 2$

(iii) $x^2 + 2x + 1$

(iv) $2x^2 \leq x \leq 2$

4. ; fn a = 2 vL b = 6 2 gS rks fuEufyf[kr oF eku Kkr dhft, %

(i) $a^2 + b^2$

(ii) $a^2 + ab + b^2$

(iii) $a^2 \leq b^2$

5. tc a = 0 vL b = 6 1 gS rks fn, gq 0; at dk oF eku Kkr dhft, %

(i) $2a + 2b$

(ii) $2a^2 + b^2 + 1$

(iii) $2a^2b + 2ab^2 + ab$

(iv) $a^2 + ab + 2$

6. bu 0; atd dks l jy dhft, rFkk buoQ eku Kkr dhft,] tc x dk eku 2 gS%
 (i) $x + 7 + 4 (x \leq 5)$ (ii) $3(x+2) + 5x \leq 7$
 (iii) $6x + 5 (x \leq 2)$ (iv) $4(2x \leq 1) + 3x + 11$
7. bu 0; atd dks l jy dhft, rFkk buoQ eku Kkr dhft,] tc $x = 3, a = \leq 1$ vks
 $b = \leq 2$ gS%
 (i) $3x \leq 5 \leq x + 9$ (ii) $2 \leq 8x + 4x + 4$
 (iii) $3a + 5 \leq 8a + 1$ (iv) $10 \leq 3b \leq 4 \leq 5b$
 (v) $2a \leq 2b \leq 4 \leq 5 + a$
8. (i) ; fn $z = 10$ gS rks $z^3 \leq 3(z \leq 10)$ dk eku Kkr dhft, %
 (ii) ; fn $p = \leq 10$ gS rks $p^2 \leq 2p \leq 100$ dk eku Kkr dhft, A
9. ; fn $x = 0$ i j $2x^2 + x \leq a$ dk eku 5 oQ cjkj gS rks a dk eku D;k gksuk pkfg, \
10. 0; atd $2(a^2 + ab) + 3 \leq ab$ dks l jy dhft, vks bl dk eku Kkr dhft,] tc
 $a = 5$ vks $b = \leq 3$ gSA

12.8 chth; 0; atdkaoQ i; kxpuL vks fu; e

ge i gysHkh nE k popQ gff fd xf.kr ea l wika (formulas) vks fu; e (rules) dks l f{klr vks 0; ki d : i eQ chth; 0; atdka dk i; kx djoQ fy[kk tk l drk gSA ge uhps vud mnkj .k n[ks%

• ifjeki l

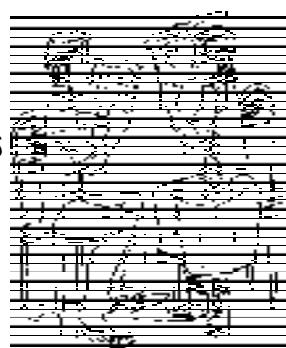
- , d l eckgqf=kHkq dk ifjeki = $3 \times ml dh Hkqk dh yekbZ gksk gSA$; fn bl l eckgq f=kHkq dh Hkqk dh yekbZ dks z l s 0; Dr djI rks ml dk ifjeki = $3l$ dk gksk A
- bl h i;dkj] , d oxz dk ifjeki = $4l$ gksk gS tgkz oxz dh Hkqk dh yEckbZ gSA
- , d l e ipHkq (regular pentagon) dk ifjeki = $5l$ gksk gS tgkz ml dh Hkqk dh yekbZ gS br; kfn A

• {kski Qy l

- ; fn ge , d oxz dh Hkqk dks z l s 0; Dr djI rks oxz dk {kski Qy = l^2 gksk gSA
- ; fn ge , d vks r dh yekbZ vks pksMkbZ dks Øe'k% l vks b l s 0; Dr djI rks vks r dk {kski Qy = $l \times b = lb$ gksk gSA
- bl h i;dkj] ; fn , d f=kHkq dk vks/kj b vks mOpkbZ h gS rks f=kHkq dk

$$\text{ {kski Qy} } = \frac{b}{2} \frac{h}{2} gksk gSA$$

, d ckj fd l h nh gQ jkf'k oQ fy, l vFkk~chth; 0; atd Kkr gks tk,] rks ml jkf'k dk eku okNir ifrcakaoQ vrxz ifjdfyr fd;k tk l drk gSA mnkj .kkFk yekbZ 3 cm dh Hkqk okys, d fn, gq oxz dk ifjeki] oxz oQ ifjeki oQ 0; atd] vFkk~4/ eI= 3 cm j [kus ij iklr fd;k tkrk gSA fn, gq oxz dk ifjeki = (4×3) cm = 12 cm



bI h i₁dkj] bI oxZ dk {kski ly} oxZ o₁ {kski ly o₁ 0; d} vFkk₁-P₁e₁ = 3 cm j[k dj i₁lr fd;k tkrk gSA

fn, gq oxZ dk {kski ly} = (3)² cm = 9 cm²

● I₁; k i₁fr: i₁a (Patterns) o₁ fy, fu; e

fuEufyf[kr dFkuak dk vè;; u dhft, %

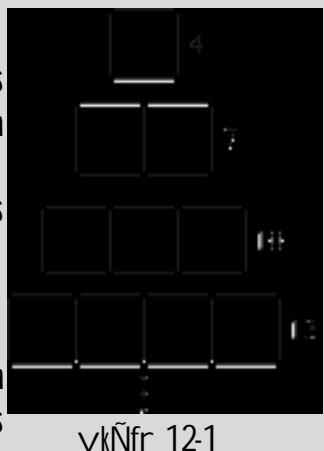
1. ;fn fdIh ikN_n I₁; k dks n ls 0; Dr fd;k tk, rks ml dk ifjorh (successor) (n + 1) gksk gSA ge bl dh tkp fdIh Hkh ikN_n I₁; k o₁ fy, dj l drs gSA mnkgj.kkFk; fn ikN_n I₁; k 10 g₁ rks ml dk ifjorh 10 + 1 = 11 g₁ tks l oofmr gS (Kkr g) A
2. ;fn fdIh ikN_n I₁; k dks n ls 0; Dr fd;k tk, rks 2n, d l e I₁; k gksk gSrFk (2n + 1), d fo"ke I₁; k gksk gSA vkb, bl dh tkp dk₁ZHkh ikN_n I₁; k ekuk 15 ysdj djA vc] 2n = 2 × 15 = 30 g₁ tks okLro e, d l e I₁; k gSrFk 2n + 1 = 2 × 15 + 1 = 30 + 1 = 31 g₁ tks okLro e, d fo"ke I₁; k gSA

blg₁ dhft,

ekfpI dh rhfy; k nk₁ i₁it djusdh lhdka; k l jdMkao₁ cjkj yckbz o₁ VdMka o₁ Nk/s j₁kk[kMka dks yhft, A mlga vknfr; kaean'kk, vuq kj i₁fr: i₁a (patterns) e₁ t kFM+ %

1. vknfr 12-1 eacu s i₁vuZ dks nf[k, A

bI ea pkj j₁kkvka l s cus vkdkj dh i₁ujkofuk gks jgh gSA t₁ k fd vki ns[k l drsgfd, d vkdkj dks cukus o₁ fy, pkj j₁kk[kMka dh vko'; drk gksk g₁ nks vkdkjka o₁ fy, 7] rhu vkdkjka o₁ fy, 10] bR; kfn j₁kk[kMka dh vko'; drk gksk gSA ;fn vkdkjka dh I₁; k n g₁ rks mlga cukus o₁ fy, vko'; d j₁kk[kMka dh I₁; k (3n + 1) gksk A vki bl dh l R; rk dh tkp n = 1, 2, 3,...,10,... bR; kfn yd₁ dj l drs gSA ;fn cuk, x, vkdkjka dh I₁; k 3 g₁ rks vko'; d j₁kk[kMka dh I₁; k 3 × 3 + 1 = 10 gksk t₁kfd vknfr ls Hkh ns[k tkI drk gSA



vknfr 12-1



vknfr 12-2

2. vc vknfr 12-2 efn, i₁vuZ j fopkj dhft, A ;gk₁vkdkj dh i₁ujkofuk gks jgh gSA vkdkjka 12]3]... dks cukus o₁ fy, vko'; d j₁kk[kMka dh I₁; k, j Øe'k%3]5]7]9]-- gSA Øe'k%; fn n cuk, x, vkdkjka dh I₁; k dks0; Dr djrk gS rks vko'; d j₁kk[kMka dh I₁; k 0; d (2n + 1) l s i₁lr gksk A 0; d l gh gS; k ugh dh tkp vki no₁ fdIh Hkh elu dks yd₁ dj l drs gSA mnkgj.kkFk n = 4 yusij] okN_n j₁kk[kMka dh I₁; k 2n + 1 = (2 × 4) + 1 = 9, gksk tks okLro e₁ 4 o₁ cukus o₁ fy, vko'; d gSA

iż kl dhft,

n'kkz x, vkl/kjHkr vklkjaks ydij mijkDr iżdkj oż iżuz cukb, %



vklkjaksukus oż fy, vko'; d jskk[1] adh I {; k nkbvlg fy[lh għgħSA l-kfkk gh-n vklkjaksukus oż fy, vko'; d jskk[1] aqob n'kkus okyk 0; atd Hkk nkbavlg fn; k għix għSA°

vklxs cfet vlgħi, h gh-vlgħ iż-żukkdh [kfst dhft, A

blgħa dhft,

vknifra eż-żikk vvu qkj] ċnugħka (dots) oż iż-żukkdh, A ; fn vki , d vlyks k dkkx (dot paper) yl rks iż-żukkdh cukuk I jy jgħix A

nf[k, fd fdl iżdkj ċnugħka dks, d oxzoż vklkj eż-żi; ofLFkr fd; k x; k għSA ; fn fd l-hof f'k "V vklkj eż-żi, d iż-żi; d Lrakk eż-żen ċnugħka dh I {; k pj n yrss għi rks vklkj eż-żi ofly ċnugħka dh I {; k 0; atd n × n = n² Is iż-żekk għix A mnkgji. lkFk n = 4 yhft, A ml- vklkj oż fy, ft l-dh iż-żi; d iż-żi; d Lrakk eż-żi 4 ċnugħi rc ofly ċnugħka dh I {; k 4 × 4 = 16 għixi] ft l-soklro eż-vknifra Is nsekk tk l-drk għSA vki bl-hiż-żi dh tkop n oż vll; eku ydżi Hkk dj l-drsgħA ikphu ; ukku xf.kkha usbu I {; kvu 1] 4] 9] 16] dks oxzi I {; kvu (square numbers) Isukċid idher fd; kā

• 1

• • 4

• • • 9

• • • • 16

• • • • • 25

• • • • • • 36

• • • • • • • • n²

• oħN vlgħi I {; k iż-żuz

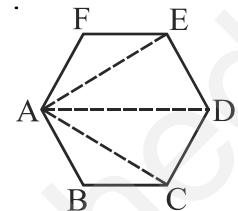
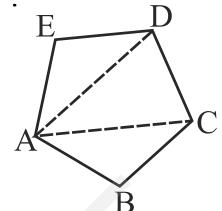
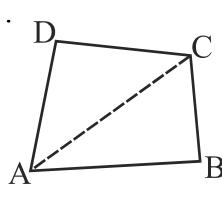
vkb, I {; kvu oż-żi, d vll; iż-żuz i jfopkji dji ft l-egejkjh l-għix; rk oż fy, dkkz vknifra cuu għo u għo għs% 3, 6, 9, 12, ..., 3n, ...

; s I {; k, j 3 oż-żekk (multiples) għiż-żi blgħa 3 Is iż-żekk djsa għi vklkjgħ Øe eż-żi; ofLFkr fd; k x; k għSA n-o LFkku iż-żi vklusokys in dks 3n Is 0; Dr fd; k x; k għbl dh l-għix; rk I {; vki I jyrik id-100 LFkku iż-żi vklusokys in (tiks 3 × 10 = 30 għi rFk 100 n-o LFkku iż-żi vklusokys in (tiks 3 × 100 = 300 għi b-R; kfn Kkr dj l-drsgħA

● T; kfefr ea i ūz

, d pr̄it oł fdlh 'kh'kz l sml oł fdrusfod. kz [khps tk l drsgs\ tlp dñft, fd budh l q;k , d gSA

, d ipHkz oł , d 'kh'kz l sml oł fdrusfod. kz [khp l drsgs\ tlp dñft, fd budh l q;k nks gSA



, d "kvHkz oł , d 'kh'kz l sml oł fdrusfod. kz [khps tk l drsgs\ tlp dñft, ;g l q;k 3 gSA

n Hkz okys fdlh cghkz oł , d 'kh'kz l sge oly (n ó 3) fod. kz [khp l drs gSA , d l IrHkz (7 Hkz,;) vks v"VHkz (8 Hkz,;) oł fy,] much vklñfr; k [khp djoł bl dh tlp dñft, A :g l q;k , d fHkz (3 Hkz,;) oł fy, D; k gS\ è;ku nhft, fd fdlh cghkz oł fdlh , d 'kh'kz l s [khps x, fod.kz ml s mrus vukfu0;kih (non-overlapping) (tks, d nñjs dks u <drs gk> f=Hkz ka ea foHkzftr djrs gñftruh fod. kke dh l q;k l svf/d 1 l q;k gksh gSA

izukoyh 12.4

1. cjkcj yekbz oł jçkk[kb/ka l scuk, x, vdkz oł i ūz dks ns[k, A vki jçkk[kb/ka l scus gq bl i dkj oł vdkz dks byDVñud ?kfM+ka ;k olyD; wjka ij ns[k l drs gSA



- (a)
 6 11 16 21 ... (5n + 1) ...
- (b)
 4 7 10 13 ... (3n + 1) ...
- (c)
 7 12 17 22 ... (5n + 2) ...

;fn cuk, x, vdk_k dh I₁;k n yh tk,] rks ml o₀ fy, vko'; d j_{kk}[k_{kk} dh (n) I₁;k n'kk_{kk} oky_k chth; 0;atd iR; d i_{kk} o₀ nkb±v_{kj} fy[k_{kk} x;k gSA

[, L, [o₀ i_{dkj} o₀ 5]100 vdk_k dks cokus o₀ fy, fdrus j_{kk}[k_{kk} dh vko'; drk g_{kk} \

2. I₁;k i_{kk} dh fuEufyf[kr I₁ kj.kh dks i_{jk} djus o₀ fy,] fn, gq chth; 0;atd_k dk i_z k_x dhft, %

Øe I ₁ ;k	0;atd	in								
		i _{gyk}	n _{jk}	r _{hj} j _k	p _{kk}	i _{kpoka}	...	n _{lok}	...	I _{kk} k _j
(i)	2n + 1	1	3	5	7	9	-	19	-	-
(ii)	3n + 2	2	5	8	11	-	-	-	-	-
(iii)	4n + 1	5	9	13	17	-	-	-	-	-
(iv)	7n + 20	27	34	41	48	-	-	-	-	-
(v)	$n^2 + 1$	2	5	10	17	-	-	-	10,001	-

geus D; k ppk_k dh \

- pjka v_{kj} vpjka l s chth; 0;atd curs gSA 0;atd_k dks cokus o₀ fy,] ge pjka v_{kj} vpjka i_j;k_x] 0;odyu] x_{ku} v_{kj} foHkktu dh I₁ f₀;k_j djrs gSA mnkgj.kkFk] 0;atd_k 4xy + 7 pjka x v_{kj} y rFk_k vpjka 4 v_{kj} 7 I₁ scuk;k x;k gSA vpj 4 rFk_k pjka x v_{kj} y dks x_{kk} djo₀ 4xy cukdj ml e₀ 7 t_{kk}+dj 4xy + 7 cuk;k tkrk gSA
- 0;atd i_{nk} l sfeydj curs gSA i_{nk} dks t_{kk}+dj 0;atd_k c_k t_{kk} gSA mnkgj.kkFk] i_{nk} 4xy v_{kj} 7 dks t_{kk} l s 0;atd 4xy + 7 cu t_{kk} gSA
- ,d i_n] x_{ku}[k_{kk} dk ,d x_{ku} l y g_{kk} gSA 0;atd 4xy + 7 e₀ i_n 4xy x_{ku}[k_{kk} x,y v_{kj} 4 dk ,d x_{ku} l y gSA pjka okys x_{ku}[k_{kk} chth; x_{ku}[k_{kk} dgykr gSA
- i_n dk x_{kk} ml dk I₁;k_x d_{kk} x_{ku}[k_{kk} g_{kk} gSA dHk_k&dHk_k i_n dk dk_{kk} H_k ,d x_{ku}[k_{kk} i_n o₀ 'k_{kk} H_k dk x_{kk} dgykr gSA
- ,d ;k vf/d i_{nk} l s c_k 0;atd ,d cg_{kk} i_n dgykr gSA fof'k"V : i₁ ,d i_n oky_k 0;atd ,d i_n h] nks i_{nk} oky_k 0;atd f₀ i_n rFk_k rhu i_{nk} oky_k 0;atd f₀ i_n dgykr gSA
- os i_n ftue_k chth; x_{ku}[k_{kk} ,d t_{kk} s g_{kk}] l_{ku} i_n dgykr g_{kk} rFk_k fHk_k & fHk_k chth; x_{ku}[k_{kk} okys i_n v_{kj} l_{ku} i_n dgykr g_{kk} gSA bl i_{dkj} 4xy v_{kj} -3xy l_{ku} i_n g_{kk} i_j r_q 4xy v_{kj} -3x l_{ku} i_n ugh gSA
- nks l_{ku} i_{nk} dk ;k_x (;k v_{rj}) ,d vU; l_{ku} i_n g_{kk} g_{kk} ft l_{dk} x_{kk} mu l_{ku} i_{nk} o₀ x_{kk} dk o₀ ;k_x (;k v_{rj}) o₀ c_{jkj} g_{kk} gSA bl i_{dkj}] 8xy ó 3xy = (8 ó 3)xy, vFk_k ~ 5xyA

8. tc ge nks cth; 0; ~~at~~ dks tkrk g~~s~~ rks l eku i nks dk] m~~o~~ij of.kr fu; e o~~l~~
vukj tkMk tkrk g~~s~~ tks l eku i n ughag~~s~~mlgao~~s~~ sgh Nkm+fn; k tkrk g~~s~~ bl idkj]
 $4x^2 + 5x \sqrt{5}$ $2x + 3$ dk ;ks $4x^2 + 7x + 3$ gSA ;gkj l eku i n $5x \sqrt{5}$ $2x + 7$ dj
 $7x$ cu tkrk g~~s~~ rFkk vI eku i nks $4x^2 \sqrt{5}$ 3 dks o~~s~~ sgh Nkm+fn; k tkrk gSA
9. , d l ehdj.k dks gy djus v~~k~~ fdlh l~~k~~ dk i~~z~~ks djus t~~s~~ h fLFkfr; kae~~e~~ ges~~a~~, d
0; ~~at~~ d dk eku Kkr djus dh vko'; drk gkrh gSA cth; 0~~a~~ td dk eku mu pjka
o~~l~~ ekuka ij fuHkj djrk g~~s~~ ftuls og cuk; k x; k gSA bl idkj] $x = 5$ o~~l~~ fy,
 $7x \approx 3$ dk eku 32, gSD; k~~d~~ $7 \times 5 \approx 3 = 32$ gSA
10. xf.kr e~~j~~ cth; 0; ~~at~~ dks dk i~~z~~ks djrs gq] fu; ek~~v~~ v~~k~~ l~~w~~ks dks l~~f~~{klr v~~k~~ 0; kid
: i eafy[~~kk~~ tkrk gSA
bl idkj] v~~k~~; r dk {kskiQy = 1b, g~~s~~ t~~g~~kj l~~v~~k; r dh yckbz rFkk b v~~k~~; r dh
pkMkbz gSA
, d l~~z~~; k i~~V~~u~~z~~ (k vu~~ø~~e) dk 0; kid (nok) in] n e~~a~~, d 0; ~~at~~ gkrk gSA bl
idkj] l~~z~~; k i~~V~~u~~z~~ 11, 21, 31, 41, . . . dk n oki in $(10n + 1)$ gSA

